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# A non-perturbative determination of $Z_V$ and $b_V$ for $O(a)$ improved quenched and unquenched Wilson fermions*

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By considering the local vector current between nucleon states and imposing charge conservation we determine, for $O(a)$ improved Wilson fermions, its renormalisation constant and quark mass improvement coefficient. The computation is performed for both quenched and two flavour unquenched fermions.

## 1. INTRODUCTION

Due to the presence of the 'Wilson term' in the lattice fermion action for Wilson fermions the discretisation errors are $O(a)$. As the gluon part of the action (sum of plaquettes) has only $O(a^2)$ errors, it is also desirable to achieve this for the fermion action. The Symanzik programme[1] allows a systematic reduction of errors to $O(a^2)$ upon including additional higher dimensional operators. The (on-shell) action is improved with a suitably tuned 'clover' term. However it is also necessary to improve each operator separately. Much work has been devoted to this topic in recent years; here we shall just concentrate on the local vector current: $V_\mu^{(q)} = \overline{q}\gamma_\mu q$. In this case just two additional operators $am_q V_\mu^{(q)}$ and $\frac{1}{2} ia\partial_\lambda T_{\mu\lambda}^{(q)}$ are required giving the $O(a)$ improved and renormalised vector current as

$$\mathcal{V}_\mu^{(q)R} = Z_V(1 + am_q b_V)(V_\mu^{(q)} + \tfrac{1}{2} iac_V \partial_\nu T_{\mu\nu}^{(q)})$$

with $T_{\mu\nu}^{(q)} = \overline{q}\sigma_{\mu\nu} q$. The second improvement operator only has an effect in *non-forward* matrix elements and will not be considered further here. The renormalisation constant $Z_V$ and improvement coefficient $b_V$ are functions of the coupling constant $g_0$ and perturbatively we have, [2], to one loop (independently of the presence of fermions),

$$\begin{aligned} Z_V(g_0) &= 1 - 0.12943 g_0^2 + \ldots, \\ b_V(g_0) &= 1 + 0.15323 g_0^2 + \ldots, \end{aligned}$$

but in presently accessible regions of $\beta \equiv 6/g_0^2$ there may be considerable deviations.

## 2. THE CONSERVED CURRENT

There is an exact symmetry of the action $q \to e^{-i\alpha} q$, $\overline{q} \to e^{i\alpha}\overline{q}$ giving via the Noether theorem the Ward identity (WI)

$$\langle \Omega \overline{\Delta}_\mu J_\mu^{(q)} \rangle = \left\langle \frac{\partial \Omega}{\partial q} q \right\rangle + \left\langle \overline{q} \frac{\partial \Omega}{\partial \overline{q}} \right\rangle,$$

where $\Omega$ is an arbitrary operator, $\overline{\Delta}_\mu$ is the lattice backward derivative, and $J_\mu^{(q)}$ the exactly con-

*Talk given by R. Horsley at Lat02, Boston, U.S.A.
[1]For a recent review see, for example, [1].

served vector current[2] ($CVC$),

$$J_\mu^{(q)}(x+\tfrac{1}{2}\hat{\mu}) = \tfrac{1}{2}\left[\overline{q}_x(\gamma_\mu-1)U_\mu(x)q_{x+\hat{\mu}} - \overline{q}_{x+\hat{\mu}}(\gamma_\mu+1)U_\mu(x)^\dagger q_x\right]$$

Roughly speaking the RHS of this equation counts the number of $q$ and $\overline{q}$ in operator $\Omega$. For our numerical results we take $\Omega \to B\overline{B}$ where $B$ is the (unpolarised) nucleon operator to give, upon solving the WI equation,

$$\begin{aligned} R(J_4^{(u-d)}) &= \frac{\langle B(t)J_4^{(u-d)}(\tau)\overline{B}(0)\rangle}{\langle B(t)\overline{B}(0)\rangle} \\ &= \begin{cases} c_1^{(u-d)} & 0<\tau<t \\ c_2^{(u-d)} & t<\tau<N_T-1 \end{cases} \end{aligned}$$

($J_4^{(u-d)} \equiv J_4^{(u)} - J_4^{(d)}$) where $c_i^{(u-d)}$ are constants and with *jump*, $\Delta R(J_4^{(u-d)}) \equiv c_1^{(u-d)} - c_2^{(u-d)} = 1$, ie charge conservation. The numerical advantage of considering $u-d$ is that the hard to compute quark line disconnected terms cancel. (For the $CVC$ this term vanishes though.) In Fig. 1 we show this ratio for the conserved vector current. A very good signal is observed (indeed the result should be true to machine accuracy).

## 3. THE LOCAL CURRENT

The local vector current ($LVC$) is not conserved on the lattice and so we do not expect the jump to be equal to one. This is shown in the RH picture in Fig. 1. We now define the renormalisation constants ($Z_V$, $b_V$) by demanding that the renormalised local current has the same behaviour as the conserved current, so that

$$Z_V(1+am_qb_V) = \left[\Delta R(V_4^{(u-d)})\right]^{-1}.$$

Thus upon plotting the data, the intercept gives $Z_V$ and the gradient $Z_Vb_V$. ($am_q = \frac{1}{2}(1/\kappa - 1/\kappa_c)$ and $\kappa_c(g_0)$ is estimated from $r_0m_q \propto (r_0m_{ps})^2$.) In Fig. 2 we show quenched results from which the intercept and gradient can be found. Other alternative non-perturbative determinations have been given by the ALPHA collaboration, [3], using the Schrödinger functional, the LANL collaboration, [4] using other Ward identities and Martinelli et al., [5] by 'mimicking' perturbation theory.

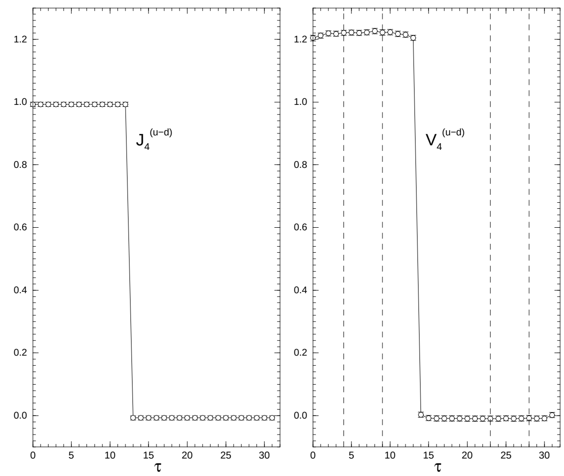


Figure 1. *$R(J_4^{(u-d)})$ and $R(V_4^{(u-d)})$ plotted against the operator position $\tau$ for the quenched ($n_f = 0$) data set $\beta = 6.0$, $\kappa = 0.1342$ on a $N_S^3 \times N_T = 16^3 \times 32$ lattice with $t = 13$. Typical fit intervals for $V_4^{(u-d)}$ are given by the pairs of vertically dashed lines.*

As well as quenched data sets ($n_f = 0$), in collaboration with UKQCD, we have also generated unquenched data sets. In this study we use the configurations with parameters given in Table 1.

| $\beta$ | $\kappa_{sea}$ | Volume | Trajs. | Group |
|---|---|---|---|---|
| 5.20 | 0.1342 | $16^3 \times 32$ | 5000 | QCDSF |
| 5.20 | 0.1350 | $16^3 \times 32$ | 8000 | UKQCD |
| 5.20 | 0.1355 | $16^3 \times 32$ | 8000 | UKQCD |
| 5.25 | 0.1346 | $16^3 \times 32$ | 2000 | QCDSF |
| 5.25 | 0.1352 | $16^3 \times 32$ | 8000 | UKQCD |
| 5.25 | 0.13575 | $24^3 \times 48$ | 1000 | QCDSF |
| 5.29 | 0.1340 | $16^3 \times 32$ | 4000 | UKQCD |
| 5.29 | 0.1350 | $16^3 \times 32$ | 5000 | QCDSF |
| 5.29 | 0.1355 | $24^3 \times 48$ | 2000 | QCDSF |

Table 1
*Data sets used in the unquenched, $n_f = 2$, simulations.*

We now present our results. In Fig. 3 we show $Z_V$ and in Fig. 4, $b_V$. For quenched fermions good agreement with other methods is found.

[2] $O(a)$ improvement for non-forward matrix elements requires, as for the local vector current, an additional operator $\frac{1}{2}ia\frac{1}{2}[\partial_\nu T_{\mu\nu}^{(q)}(x) + \partial_\nu T_{\mu\nu}^{(q)}(x+\hat{\mu})]$.

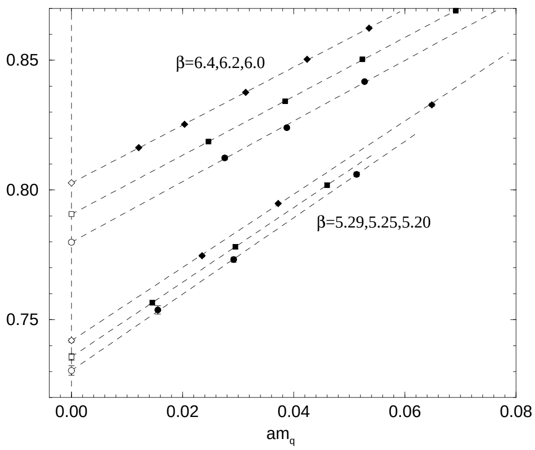


Figure 2. $\Delta R(V_4^{(u-d)})$ *for quenched configurations for* $\beta = 6.4$, $6.2$ *and* $6.0$ *(upper set of curves, top to bottom respectively) and for the unquenched configurations for* $\beta = 5.29$, $5.25$ *and* $5.20$ *(lower set of curves).*

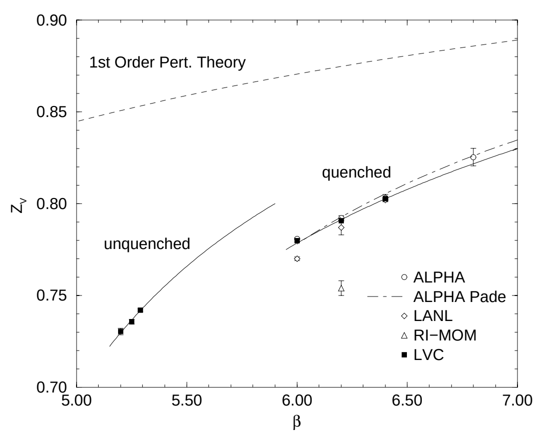


Figure 3. $Z_V$ *(LVC, filled squares) determined in this work for quenched and unquenched* $O(a)$ *improved fermions. For the quenched case a comparison is made with ALPHA, [3], LANL, [4] and RI-MOM, [5]. Padé fits are also given for our and the ALPHA results.*

## 4. CONCLUSIONS

The method described here reproduces the results of other approaches for $O(a)$ improved quenched fermions. (But one needs to remember that $Z_V$ definitions can vary by $O(a^2)$ while $b_V$ definitions may vary by $O(a)$.) For $O(a)$ improved unquenched fermions $Z_V$ is *smaller* and $b_V$ *larger* than for quenched fermions at the same lattice spacing (roughly $a_{n_f=2}(5.25) \sim a_{n_f=0}(6.0)$). Further details and final results will appear in [6].

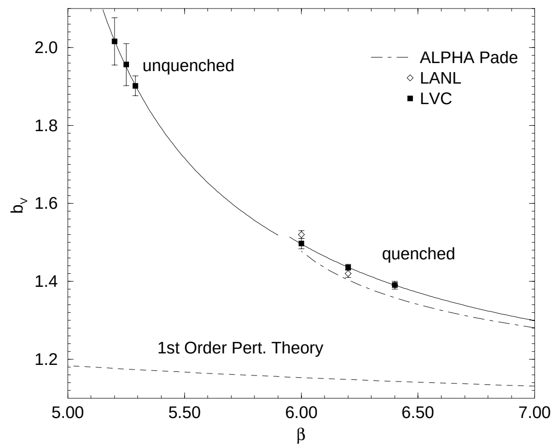


Figure 4. $b_V$ *(LVC, filled squares) determined in this work again for both quenched and unquenched* $O(a)$ *improved fermions. Also shown is the Padè fit from ALPHA, [3], and the LANL, [4] results for quenched fermions.*

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